Section 4.7 Two Independent Samples Wilcoxon

1. mod4.7

1.1 Two Independent Samples Wilcoxon

Notes:

2. This module contains 2 main sections: hypothesis testing for 2 or more independent groups, and use of non-parametric methods.
1.2 Learning Outcomes

Notes:

1. Here are the Module 4 learning outcomes.
2. We will use SPSS for some of these objectives, but mostly hand calculations that are done as practice exercises.
3. You will notice that hypothesis tests will be conducted and calculated using a range of different study designs.
1.3 Two Independent Samples: Wilcoxon Rank Sum Test

Notes:

1. Here is the framework for non-parametric hypothesis testing for 2 independent groups.

2. We will conduct the Wilcoxon Rank Sum test for which we just reviewed the results in comparison to the parametric t-test.
1.4 Two Independent Samples: Wilcoxon Rank Sum Test

**Notes:**

1. Here we are using data from our previously example where group 2 tended to have higher values than group 1, but group 1 had one extreme value.

2. Note that we have ordered and rank each value for the 2 groups, and then summed the ranks for the 2 groups denoted as R1 and R2.

3. As seen, group 2 has a much greater sum with R2=145 compared to group 1 where R1=65.1.
1.5 Two Independent Samples: Wilcoxon Rank Sum Test

Notes:

1. Using the values for R1 and R2, we now calculate the Wilcoxon Rank Sum test.
2. Note that the test statistic, U, is evaluated by the smaller value of U1 and U2. In this case, thus is the U2 value of 10.0.
1.6 Two Independent Samples: Wilcoxon Rank Sum Test

Notes:

1. Now we evaluate the U value of 10.0 using our standard 5-step approach for hypothesis testing.
2. Refer to table 5 of the appendix in your textbook.
3. You will note that with sample size of 10 in each group (n1 and n2), the critical value for a 2-sided test with type I error rate of 0.05 is 23.
### 1.7 Practice Exercise

**Example:** Consider a trial of a new drug to reduce symptoms of asthma in children (episodes of shortness of breath); one-sided; \( \alpha = 0.05 \)

<table>
<thead>
<tr>
<th>Group</th>
<th>Placebo</th>
<th>Drug</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group</th>
<th>Placebo</th>
<th>Drug</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Sum of Ranks: \( R_1 = \) \( n_1 \) \( n_2 \)

**Notes:**

1. **Now it is time to practice.**
2. **Using your handout for this module, complete the entries for non-parametric hypothesis testing for 2 independent groups and by use of the Wilcoxon Rank Sum test.**
3. **Complete the middle column of ordered values first, followed by the Ranks column.**
1.8 Practice Exercise Answers

Example: Consider a trial of a new drug to reduce symptoms of asthma in children (episodes of shortness of breath); one-sided; α=0.05

<table>
<thead>
<tr>
<th>Group</th>
<th>Drug</th>
<th>Mucus</th>
<th>Drug</th>
<th>Mucus</th>
<th>Ranks</th>
</tr>
</thead>
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<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
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<td>6</td>
<td>4</td>
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<td>9</td>
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<td>10</td>
<td>20</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

\[
\begin{align*}
U_1 &= \frac{n_1(n_1+1)}{2} - R_1 = 5(5) + 3 = 37 - 37 = 37 \\
U_2 &= \frac{n_2(n_2+1)}{2} - R_2 = 5(5) + 3 = 25 + 3 = 28 \\
\end{align*}
\]

Notes:

1. Here are the results.
2. Note that there appears to be a difference in the sum of ranks with \( R_1 = 37 \) and \( R_2 = 18 \).
3. The lower of the 2 “U” values for hypothesis testing is group 1 with \( U_1 = 3 \).
1.9 Practice Exercise

Notes:

1. Continuing on with the practice exercise.
2. Using your handout for this module, complete the entries for non-parametric hypothesis testing for 2 independent groups and by use of the Wilcoxon Rank Sum test.
3. Remember that you reject the null hypothesis if the U value is “less than or equal” to the critical value.
1.10 Practice Exercise Answers

1. Set up the hypothesis and level of significance:
   $H_0$: The two populations are equal
   $H_1$: The two populations are not equal $\alpha = 0.05$

2. Select the appropriate test statistic:
   Mann-Whitney $U$-test

3. Set up the decision rule:
   Reject $H_0$ if $U \leq 4$ (see Table 5)

4. Compute the test statistic:
   From previous slide:
   $U = \min(R_1, R_2)$ $U = 3$

5. Conclusion:
   Reject $H_0$ because $3(U) < 4$ (critical value) (populations are not equal)

Notes:

1. We evaluate the U value of 3.0 using our standard 5-step approach for hypothesis testing.
2. Refer to table 5 of the appendix in your textbook.
3. You will note that with sample size of 5 in each group (n1 and n2), the critical value for a 2-sided test with type I error rate of 0.05 is 4.
1.11 Conclusion