Section 4.2 Two-Sample: Independent Groups - Continuous Outcome

1. mod4.2

1.1 Two-Sample: Independent Groups - Continuous Outcome

Notes:

1. Let's review hypothesis testing for a 2-sample independent groups design, and comparison of a continuous outcome variable.
2. We will practice calculations and also use SPSS.
1.2 Learning Outcomes

Notes:

1. Here are the Module 4 learning outcomes.
2. We will use SPSS for some of these objectives, but mostly hand calculations that are done as practice exercises.
3. You will notice that hypothesis tests will be conducted and calculated using a range of different study designs.
1.3 Two-Sample: Independent Groups - Continuous Outcome

Notes:

1. Let's walk through calculation of hypothesis testing for a 2-sample independent group design for a continuous outcome variable.
2. Note the different parameters, z and t, to be calculated based on the sample size.
1.4 Two-Sample: Independent Groups - Continuous Outcome Interaction

Notes:

Example calculation of hypothesis testing for a 2-sample independent group design with continuous outcome variable

Step 2, for the appropriate test statistic, we use \( z \) instead of \( t \) because the sample sizes are larger than 30.

Step 3, set up the decision rule.

Step 4, compute the test statistic.

Step 5, for the conclusion, we reject the null hypothesis, meaning that the 2 group means are statistically different.
Tab1 (Slide Layer)

Step 1:
Set up the hypothesis and determine the level of statistical significance (including 1 versus 2-sided hypothesis)

\[ H_0: \mu_1 = \mu_2 \]
\[ H_1: \mu_1 = \mu_2 \text{ (two-sided hypothesis)} \]
\[ \alpha = 0.05 \]

Tab3 (Slide Layer)

Step 3:
Set up the decision rule:

Reject \( H_0 \) if \( z \leq 1.96 \) or \( z \geq 1.96 \)
Tab4 (Slide Layer)

Step 4:
Compute the test statistic
\[ S_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \]
\[ S_p = \frac{(162 - 1)(17.5)^2 + (191 - 1)(20.1)^2}{162 + 191 - 2} \]
\[ = \sqrt{359.12} = 19.0 \]

\[ z = \frac{\bar{X}_1 - \bar{X}_2}{S_p \sqrt{1/n_1 + 1/n_2}} \]
\[ z = \frac{128.2 - 126.5}{19.0 \sqrt{1/1623 + 1/1911}} \]
\[ = 2.66 \]

Tab5 (Slide Layer)

Step 5:
Conclusion

Reject H_0 because 2.66 > 1.96
1.5 Practice Exercise Part 1

Example: From the Heart SCORE Study, compare mean total cholesterol levels between men and women. (α = 0.05)

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>x</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>165</td>
<td>198.88</td>
<td>38.416</td>
</tr>
<tr>
<td>Women</td>
<td>317</td>
<td>227.22</td>
<td>42.093</td>
</tr>
</tbody>
</table>

1) Set up the hypothesis and determine the level of statistical significance (including 1 versus 2-sided hypothesis).

H₀: ____________________
H₁: ____________________

2) Select the appropriate test statistic:

n₁ > 30 and n₂ > 30, so use z

3) Set up the decision rule: ________________

Notes:

1. Now it is time to practice.
2. Using your handout for this module, complete the entries for hypothesis testing of a continuous outcome and 2 independent groups.
1.6 Practice Exercise Part 1 Answers

**Example:** From the Heart SCORE Study, compare mean total cholesterol levels between men and women. \((\alpha = 0.05)\)

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<td>337</td>
<td>222.23</td>
<td>62.033</td>
</tr>
</tbody>
</table>

1) Set up the hypothesis and determine the level of statistical significance **(including 1 versus 2-sided hypothesis)**.

- \(H_0: \mu_1 = \mu_2\)
- \(H_1: \mu_1 \neq \mu_2\) (two-sided hypothesis)

2) Select the appropriate test statistic:
- \(n_1 > 30\) and \(n_2 > 30\), so use \(z\)

3) Set up the decision rule:
- Reject \(H_0\) if \(z \leq 1.96\) or \(z \geq 1.96\)

**Notes:**

1. Note the use of \(z\) for “large sample” testing.
2. The critical value of 1.96 is based on the 2-sided type I error rate of 0.05.
### Practice Exercise Part 2 Answers

**Example:** From the Heart SCORE Study, compare mean total cholesterol levels between men and women. \((\alpha = 0.05)\)

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1. Set up the hypothesis and determine the level of statistical significance (including 1 versus 2-sided hypothesis).
   - \(H_0: \mu_1 = \mu_2\)
   - \(H_1: \mu_1 \neq \mu_2\) (two-sided hypothesis)

2. Select the appropriate test statistic:
   \(n_1 > 30\) and \(n_2 > 30\), so use \(z\)

3. Set up the decision rule:
   Reject \(H_0\) if \(z < 1.96\) or \(z > 1.96\)

\[
z = \frac{\bar{X}_1 - \bar{X}_2}{S_p \sqrt{1/n_1 + 1/n_2}}
\]

### 1.7 Practice Exercise Part 2

**Practice Exercise Part 2**

**Example:** From the Heart SCORE Study, compare mean total cholesterol levels between men and women. \((\alpha = 0.05)\)

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<tr>
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<td>337</td>
<td>222.23</td>
<td>42.023</td>
</tr>
</tbody>
</table>

4. Compute the test statistic

\[
S_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}
\]

\[
z = \frac{\bar{X}_1 - \bar{X}_2}{S_p \sqrt{1/n_1 + 1/n_2}}
\]

5. Conclusion:

**Notes:**

1. Continuing on with the practice exercise.
2. Using your handout for this module, complete the entries for hypothesis testing of a continuous outcome and 2 independent groups.
3. Note that we are using a pooled standard deviation.
1.8 Practice Exercise Part 2 Answers

Example: From the Heart SCORE Study, compare mean total cholesterol levels between men and women. (α = 0.05)

4) Compute the test statistic

\[ s_p = \sqrt{\frac{(165-1)(38.416)^2 + (337-1)(41.023)^2}{165 + 337 - 2}} = 40.875 \]

\[ z = \frac{198.88 - 222.23}{40.875 \sqrt{\frac{1}{165} + \frac{1}{337}}} = -23.35 \cdot \frac{1}{3.884} = -6.01 \]

5) Conclusion: Reject H0: abs(-6.01) > 1.96

Notes:

1. Note that the pooled standard deviation of 40.875 is “between” the group standard deviations of 38.4 and 42.0.
2. With 2-sided testing, we reject the null hypothesis of the group means being the same.
SP Formula (Slide Layer)

Practice Exercise Part 2 Answers

Example: From the Heart SCORE Study, compare mean total cholesterol levels between men and women. (α = 0.05)

4) Compute the test statistic

\[ S_p = \sqrt{\frac{(165-1)(38.416)^2 + (337-1)(42.023)^2}{165 + 337 - 2}} = 40.875 \]

\[ z = \frac{198.88 - 222.23}{\sqrt{40.875 \left( \frac{1}{165} + \frac{1}{337} \right)}} = \frac{-23.35}{3.884} = -6.01 \]

5) Conclusion: Reject H0: abs(-6.01) > 1.96

View Sample Data (Slide Layer)

Practice Exercise Part 2 Answers

Example: From the Heart SCORE Study, compare mean total cholesterol levels between men and women. (α = 0.05)

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\[ S_p = \sqrt{\frac{(165-1)(38.416)^2 + (337-1)(42.023)^2}{165 + 337 - 2}} = 40.875 \]

\[ z = \frac{198.88 - 222.23}{\sqrt{40.875 \left( \frac{1}{165} + \frac{1}{337} \right)}} = \frac{-23.35}{3.884} = -6.01 \]

5) Conclusion: Reject H0: abs(-6.01) > 1.96
1.9 Two-Sample: Independent Groups-Continuous Outcome (Practice)

Example:
From the Heart SCORE Study, compare mean total cholesterol levels between men and women. (α = 0.05)

SPSS
Analyze
  ▶ Compare Means
    ▶ Independent Samples T-Test
      ▶ Test Variable: Total cholesterol
      ▶ Group Variable: Gender (defined as 1,2)
      ▶ Options: 95% C.I.

Notes:
1. Here are the interactive steps used in SPSS to compare a continuous outcome variable based on a 2-sample independent group design.
1.10 Conclusion

End of Section 4.2