Section 3.10 Statistical Power - Type 1 and Type II Error and Factors that Impact Statistical Power

1. Module1.3

1.1 Section 3.10 Statistical Power

Notes:

1. Now it is time to review the concept of statistical power.
2. Type 1 and Type II error and factors that impact statistical power
3. This is important for grant submissions as grant reviewers will want to make sure that your proposed study has a large enough sample size (i.e. statistical power) to draw meaningful conclusions.
1.2 Learning Outcomes

Notes:

1. Here are the module learning objectives.
2. Remember, the course evaluation you get to fill out will ask whether the learning objectives were appropriately covered and presented.
1.3 Interpreting Results

Notes:

1. Let’s review the 4 possible outcomes of any study with respect to the truth, meaning had we been able to study everyone, and without error.
2. As you can see, there are 2 possible ways to arrive at a correct conclusion, and 2 possible ways to arrive at an incorrect conclusion.
1.4 True Difference versus Random Variation

When evaluating the incidence of disease between the exposed and non-exposed groups, we need guidelines to help determine whether there is a true difference between the two groups, or perhaps just random variation from the study sample.

Notes:

1. To minimize the likelihood of making an error in our conclusions (i.e. type I or type II error), we use some general guidelines which are at the discretion of the investigator.

1.5 Fixed Alpha Level - Type 1 Error

“Conventional” Guidelines:
Set the fixed alpha level (Type I error) to 0.05.
This means, if the null hypothesis is true, the probability of incorrectly rejecting it is 5% or less.
The “p-value” is a measure of the compatibility of the data and the null hypothesis.

<table>
<thead>
<tr>
<th>DECISION</th>
<th>H₀ True</th>
<th>H₁ True</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do not reject H₀ (not stat. sig.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reject H₀ (stat. sig.)</td>
<td>Type I (alpha error)</td>
<td></td>
</tr>
</tbody>
</table>
Notes:

1. By convention (but not always), the type I error rate (known as alpha level) is set to 0.05.
2. This means we want a 5% chance or less of erroneously rejecting the null hypothesis of no association when the null hypothesis is in fact true.

1.6 Compatibility with Null Hypothesis

<table>
<thead>
<tr>
<th></th>
<th>D+</th>
<th>D-</th>
</tr>
</thead>
<tbody>
<tr>
<td>E+</td>
<td>15</td>
<td>85</td>
</tr>
<tr>
<td>E-</td>
<td>10</td>
<td>90</td>
</tr>
</tbody>
</table>

\[ \text{RR} = \text{I}_{+}/\text{I}_{-} = 1.5, \quad p = 0.30 \]

Although it appears that the incidence of disease may be higher in the exposed than in the non-exposed (RR=1.5), the p-value of 0.30 exceeds the fixed alpha level of 0.05. This means that the observed data are relatively compatible with the null hypothesis. Thus, we do not reject \( H_0 \) in favor of \( H_1 \) (alternative hypothesis).

Notes:

1. In this example, a relative risk of 1.5 is observed, which is quite a bit different from the null value of 1.0.
2. However, the p-value of 0.30 means, given the sample size, there is a 30% chance that this degree of departure (i.e. 1.5 versus null value of 1.0) could be due simply to chance or random variation.
1.7 Interpreting Results

**Fixed Beta Level - Type 2 Error**

**Conventional Guidelines:**
- Set the fixed beta level (Type II error) to 0.20.
- This means, if the null hypothesis is false, the probability of not rejecting it is 20% or less.
- The “power” of a study is (1 - beta). This means having 80% probability to reject $H_0$ when $H_1$ is true.

<table>
<thead>
<tr>
<th>DECISION</th>
<th>$H_0$ True</th>
<th>$H_1$ True</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do not reject $H_0$ (not stat. sig.)</td>
<td></td>
<td>Type II (beta error)</td>
</tr>
<tr>
<td>Reject $H_0$ (stat. sig.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

1. By convention (but not always), the type II error rate (known as beta level) is set to 0.20. Another common type II error rate is 0.10.
2. For 0.20, this means we want a 20% chance or less of erroneously accepting the null hypothesis of no association when the alternative hypothesis is in fact true.
3. Note that “statistical power” is 1 minus the type II error rate.
1.8 Statistical Power

Example:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>E^-</td>
<td>200</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>E^+</td>
<td>200</td>
<td>0.18</td>
<td>0.21</td>
<td>0.24</td>
</tr>
<tr>
<td>RR</td>
<td></td>
<td>1.8</td>
<td>2.1</td>
<td>2.4</td>
</tr>
<tr>
<td>Power</td>
<td></td>
<td>58%</td>
<td>82%</td>
<td>95%</td>
</tr>
</tbody>
</table>

With the above sample size of 400, and if the alternative hypothesis is true, we need to expect a RR of about 2.1 (power = 82%) or higher to be able to reject the null hypothesis in favor of the alternative hypothesis.

Notes:

1. This table illustrates that given the same sample size, the power to reject the null hypothesis of no association is higher as the relative risk (RR) departs from the null value of 1.0.

1.9 Factors that Affect Statistical Power

Factors that affect the power of a study:

1. The fixed alpha level (the lower the level, the lower the power).
2. The total and within group sample sizes (the smaller the sample size, the lower the power -- unbalanced groups have lower power than balanced groups).
3. The anticipated effect size (the higher the expected/observed effect size, the higher the power).
Notes:

1. Here are factors that affect statistical power, in other words, the ability to reject the null hypothesis of no association.

1.10 Type 1 versus Type 2 Error

Notes:

1. Unfortunately, the type I and type II errors work in opposite directions.

2. That is, if we lower the chance for a type I error, it increases the chance for a type II error.
1.11 Interpreting Results

**Selection of Error Rates**

**Trade-offs between fixed alpha and beta levels:**

Increasing the fixed alpha level (e.g. to < 0.10) reduces the probability of a type II error (failing to reject $H_0$ when $H_1$ is true), but at the expense of increasing the probability of a type I error if the null hypothesis is true.

**Notes:**

1. Unfortunately, the type I and type II errors work in opposite directions.
2. That is, if we increase the chance for a type I error, it lowers the chance for a type II error.
1.12 **Statistical Power and Sample Size**

<table>
<thead>
<tr>
<th>Exposure</th>
<th>N</th>
<th>Incid.</th>
<th>Risk ratio</th>
<th>P-value</th>
<th>Power</th>
<th>RR</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>1000</td>
<td>0.10</td>
<td>1.0</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Low</td>
<td>500</td>
<td>0.15</td>
<td>1.5</td>
<td>0.006</td>
<td>77%</td>
<td>1.52</td>
</tr>
<tr>
<td>Medium</td>
<td>250</td>
<td>0.15</td>
<td>1.5</td>
<td>0.02</td>
<td>60%</td>
<td>1.64</td>
</tr>
<tr>
<td>High</td>
<td>100</td>
<td>0.15</td>
<td>1.5</td>
<td>0.12</td>
<td>27%</td>
<td>2.08</td>
</tr>
</tbody>
</table>

* Power with given sample size and risk ratio (α = 0.05)
** Risk ratio needed for 80% power with given sample size

**Notes:**

1. This table indicates that given the same risk ratio (i.e. 1.5 in this example), a larger sample size translates to a lower p-value and higher statistical power.
2. This is a primary reason that research studies need to enroll a sufficient number of participants; i.e. in order to be able to make conclusive statements about the results.
1.13 Conclusion

End of Section 3.10